## Re-Exam

Autumn 2018

Important: Please make sure that you answer all questions and that you properly explain your answers. For each step write the general formula (where relevant) and explain what you do. Not only the numerical answer. If you make a calculation mistake in one of the earlier sub-questions, you can only get points for the following subquestions if the formula and the explanations are correct!

1. Short questions.
(a) Order the solution concepts based on their strength from weakest to strongest: IESDS, Nash Equilibrium, Perfect Baysian Equilibrium, Subgame Perfect Nash Equilibrium
(b) "Cheap-talk is never useful because it doesn't affect the players outcomes" True or False? What are the three requirements for cheap talk to be informative? Explain in 2-3 sentences.
(c) Consider a third-price auction of a single object (the highest bidder gets the object and pays the third highest bid; the others do not pay). Give an example (values and bids) that shows that truthful bidding is not a dominant strategy in this auction, and explain your example in 2-3 sentences.
(d) It can happen that there is no core in a coalition game. True or False?
(e) In many games the order of play matters. Give an example of a game with a first-mover advantage and a game with a last mover advantage and explain your reasoning in 2-3 sentences.
2. Consider the following game

|  |  | Player 2 |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | X |  |  |  |
| Player |  | Y | Z |  |
|  | A | 4,4 | 0,2 | 4,5 |
|  | B | $0,-1$ | 3,3 | 3,2 |
|  | C | 3,3 | $-1,1$ | 1,4 |
|  |  |  |  |  |

(a) Find all Nash Equilibria (pure and mixed) of G. Let $\mathrm{p}=$ probability that the row player plays A. $\mathrm{q}=$ probability that the column player plays Y .
(b) In which of the equilibria you found in a) does player 1 have the highest expected payoff?
(c) Use iterated elimination of strictly dominated strategies on $G$ and check that all pure strategy NE you found in a) survived IESDS. Is it possible that a NE does not survive IESDS? Explain briefly.
3. Consider the infinitely repeated game with a discount factor $\delta$, where the stage game ( G ) looks like this:

Player 2

(a) What is the Nash equilibrium of the stage game (G)? What are the players' payoffs in the NE? Could the players do better by coordinating on a Nash threat (trigger) strategy? Under what condition would they do better?
(b) Write down the simplest possible trigger strategy, for each player, such that the following holds: if both players stick to their trigger strategies, then the outcome will be ( $\mathrm{U}, \mathrm{L}$ ) in every period. Show that for sufficiently patient players, the strategy profile where both players play trigger strategies constitutes an SPNE. What is the minimum value of delta for which this strategy profile is an equilibrium?
4. Two firms compete in the market for gløgg, where Firm 1 produces red wine based gløgg and Firm 2 produces white wine based gløgg. These products are imperfect substitutes: demand for red wine gløgg is: $q_{1}=1-p_{1}+p_{2}$, and demand for white wine is $q_{2}=1-p_{2}+p_{1}$, where $p_{1}$ and $p_{2}$ are the prices of Firm 1 and 2 . Firm 1 has access to new wine barrels that allows it to produce at zero marginal cost. Firm 2 is still using old-fashioned barrels, so it has marginal costs of $c>0$. This means profits for Firm 1 are $\pi_{1}=q_{1} p_{1}$, and profits for Firm 2 are $\pi_{2}=q_{2}\left(p_{2}-c\right)$. Firms set prices simultaneously and independently.
(a) Show that in the Nash Equilibrium of this game, firms set prices $p_{1}=1+c / 3, p_{2}=$ $1+2 c / 3$. Explain intuitively why both equilibrium prices are increasing in c. (2-3 sentences).
(b) Suppose Firm 2 develops an innovation that it hopes will lower production costs. If the innovation works, then Firm 2's marginal costs become zero. If the innovation does not work, then its marginal costs remain at c. Firm 2 knows whether or not the innovation works (so it knows its own marginal cost), but Firm 1 does not. Firm 1 believes there is a probability $1 / 2$ that the innovation works, so that Firm 2 has marginal cost of zero, and a probability $1 / 2$ that it does not work, so that Firm 2 has marginal cost of c. Write down the three best-response functions that, taken together, implicitly define the prices in the Bayes-Nash equilibrium of this game. (Bonus points: Solve for the equilibrium prices.)
(c) Imagine that before setting prices, Firm 2 announces to Firm 1: "Unfortunately, the innovation does not work, so I have high costs!" Why might Firm 2 make such an announcement? How do you expect this announcement to affect the price set by Firm 1? What might Firm 2 do instead, to convince Firm 1 that it has high costs? Explain your answers briefly ( $2-3$ sentences each).
5. Have a look at the following signaling game G"

(a) Is this a game of complete or incomplete information?
(b) Find all separating Perfect Bayesian Equilibria.

